From "arrangement of work" to programming -- telling the short history of numerical analysis in computer science courses --

Gerard Alberts

paper presented at the workshop
Using History to Teach Computer Science More Efficiently
held in Amherst (MA), August 6-7, 2001

abstract
Computer pioneers in the 1940's an 1950's considered numerical analysis to be their emerging new discipline. By their very achievements, the advent of reliable machinery, numerical analysis exploded -ending one of the shortest lived disciplines in the history of science.
Still, a look at the practices of computing in those years may both reveal some of the persistent essentials of computer science and make us aware of some "forgotten" self-evidentials of our discipline. Return to Douglas Hartree and find spice for your didactics as well as resources for critical reflection.

the author
Gerard Alberts
is director of the Science and Society program at KUN, University of Nijmegen, and researcher at CWI, Center for Mathematics and Computer Science in Amsterdam

address
Dr. G. Alberts
CWI
P.O. Box 94079
nl-1090 GB Amsterdam, The Netherlands
+31.20.5924231
G.Alberts@cwi.nl

Numerical Analysis

During the interbellum Numerical Analysis gradually emerged from "applied theory", foremost in theoretical physics, aero- and hydrodynamics and applied mechanics.

Douglas R. Hartree (1897-1958) was the key figure in *Numerical Analysis* as a discipline

NUMERICAL ANALYSIS

D. R. HARTREE

SECOND EDITION

OXFORD · AT THE CLARENDON PRESS

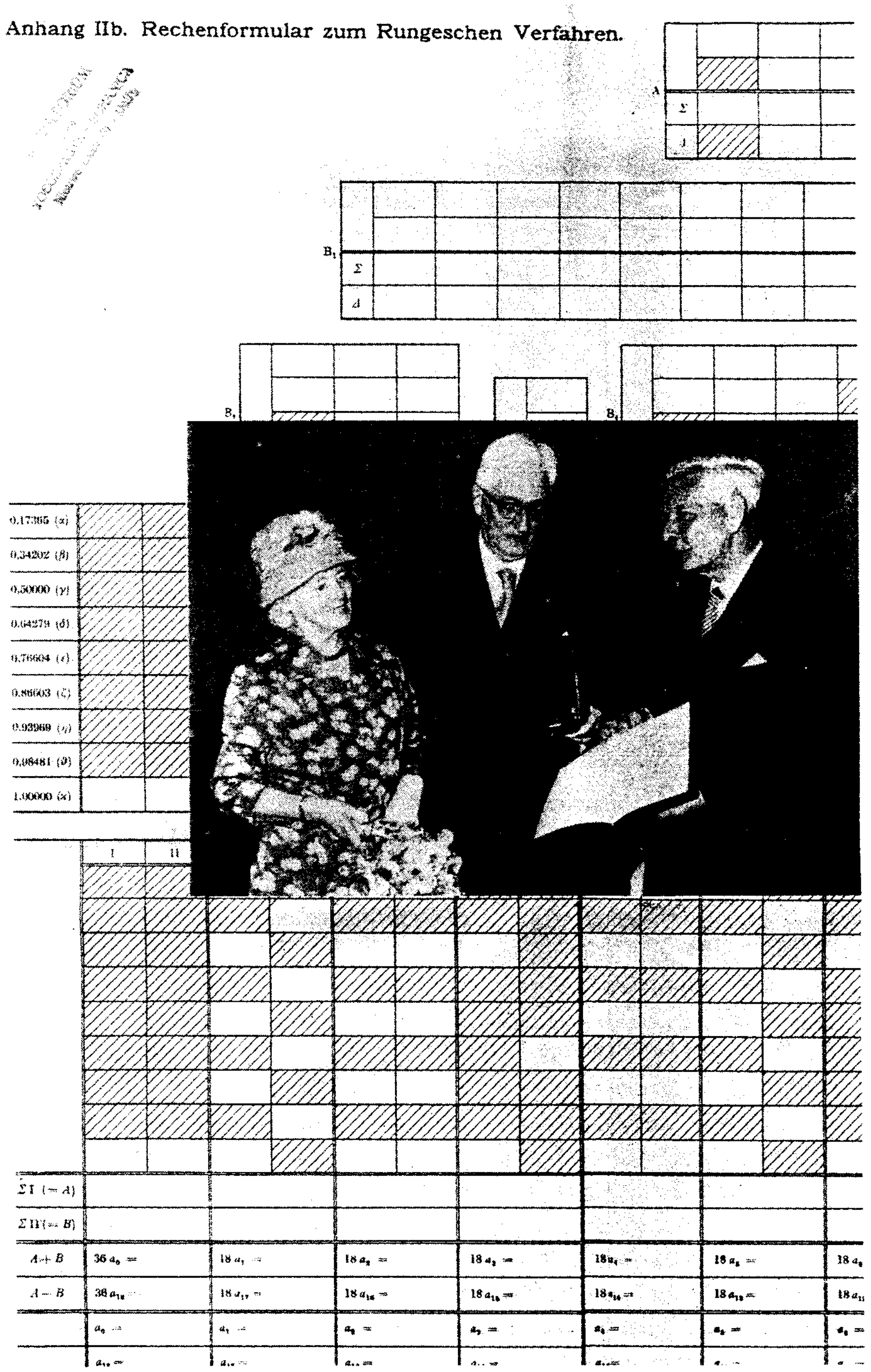
Daring reckoning

Typical of Numerical Analysis were the daring large scale computations by hand or electromechanical machines. Such endeavours were local specialties

Jan M. Burgers (1895-1981) professor of aerodynamics in Delft (1918-1955) and later in Maryland (1955-1965), also known for the Burgers Equation, had the wit, patience and endurance to perform such calculations on such problems as pumps, windmills, and aerofoil theory

Schedules

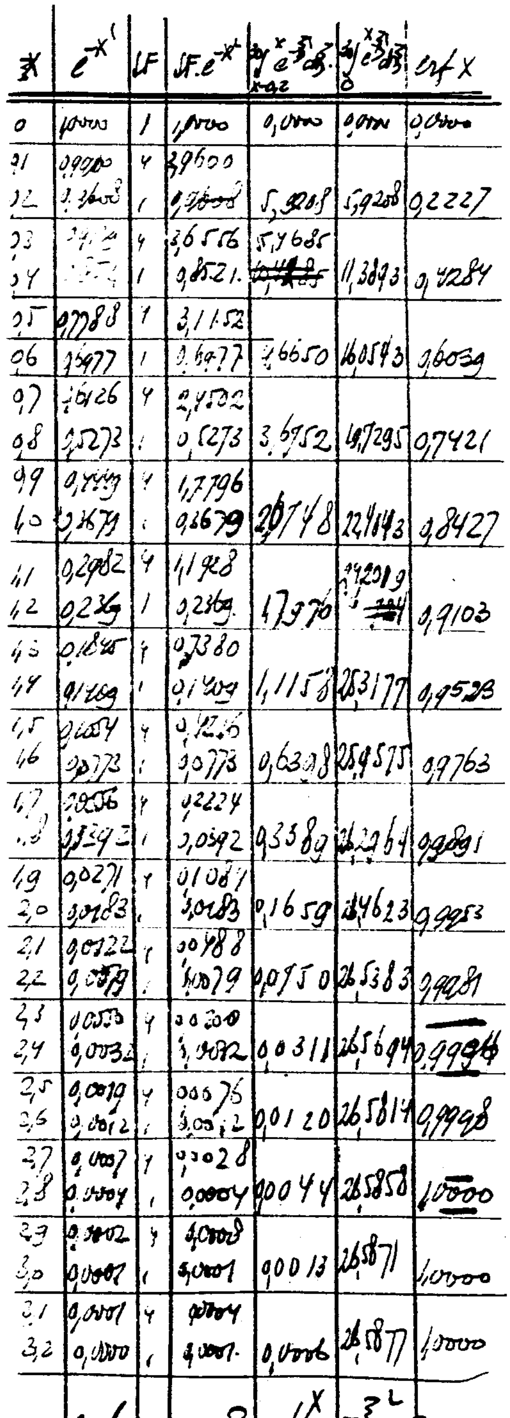
Cornelis B. Biezeno (1888-1975) was professor of applied mechanics in Delft (1914-1958). With ***R. Grammel*** he wrote the standard work *Technische Dynamik*, giving an overview of all kinds of methods to calculate solutions of technical problems.
The numerical work was clearly laid out in schedules.



Tabling

Aad van Wijngaarden (1916-1987) was a pupil of both Burgers and Biezeno. This founding father of computer science in The Netherlands is best known for the design of ALGOL 68.

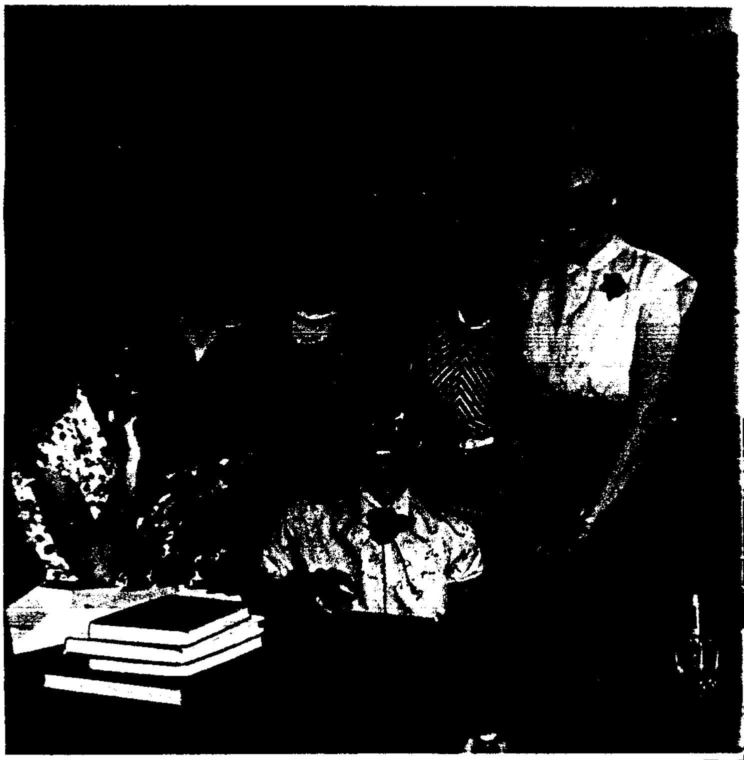
Most any problem involved tabling of approximating functions for long series of numerical values. Van Wijngaarden did such work in Delft and as head of the computing department of the Mathematical Center in Amsterdam form 1947. Characteristic of the better computers was that they undertook to develop their own calculation schedules.



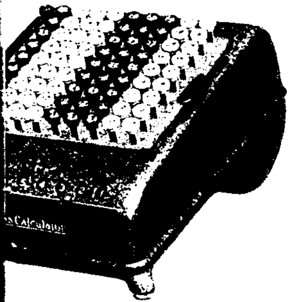
x	e^{-x^2}	SF	SF.e^{-x^2}	$\int_{x-0,2}^{x} e^{-\xi^2} d\xi$	$\int_{0}^{x} e^{-\xi^2} d\xi$	erf x
0	1,0000	1	1,0000	0,0000	0,0000	0,0000
0,1	0,9900	4	3,9600			
0,2	[illegible]	1	[illegible]	5,9208	5,9208	0,2227
0,3	[illegible]	4	3,6556	5,4685		
0,4	[illegible]	1	0,8521	[illegible]	11,3893	0,4284
0,5	0,7788	4	3,1152			
0,6	0,6977	1	0,6977	4,6650	16,0543	0,6039
0,7	0,6126	4	2,4502			
0,8	0,5273	1	0,5273	3,6752	19,7295	0,7421
0,9	0,4449	4	1,7796			
1,0	0,3679	1	0,3679	2,0748	22,4843	0,8427
1,1	0,2982	4	1,1928		24,2019	
1,2	0,2369	1	0,2369	1,7370	[illegible]	0,9103
1,3	0,1845	4	0,7380			
1,4	0,1409	1	0,1409	1,1158	25,3177	0,9523
1,5	0,1054	4	0,4216			
1,6	0,0773	1	0,0773	0,6398	25,9575	0,9763
1,7	0,0556	4	0,2224			
1,8	0,0392	1	0,0392	0,3389	26,2964	0,9891
1,9	0,0271	4	0,1087			
2,0	0,0183	1	0,0183	0,1659	26,4623	0,9953
2,1	0,0122	4	0,0488			
2,2	0,0079	1	0,0079	0,0750	26,5383	0,9981
2,3	0,0050	4	0,0200			
2,4	0,0032	1	0,0032	0,0311	26,5694	0,9993
2,5	0,0019	4	0,0076			
2,6	0,0012	1	0,0012	0,0120	26,5814	0,9998
2,7	0,0007	4	0,0028			
2,8	0,0004	1	0,0004	0,0044	26,5858	1,0000
2,9	0,0002	4	0,0008			
3,0	0,0001	1	0,0001	0,0013	26,5871	1,0000
3,1	0,0001	4	0,0004			
3,2	0,0000	1	0,0001	0,0006	26,5877	1,0000

The Work and the Girls

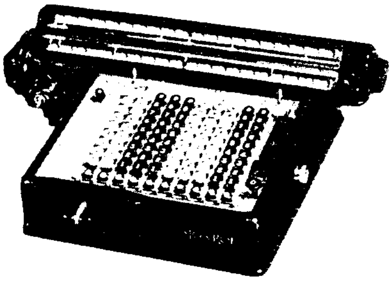
The large scale computing was done by human computers, girls: at the Mathematical Center by "Van Wijngaarden's girls", ***Truus Hurts***, ***Loes Kaarsemaker***, ***Reina Mulder*** and others
They would use Marchant and Monroe electrical devices



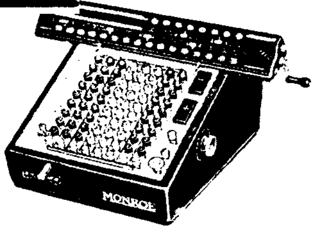
CALCULATING MACHINES



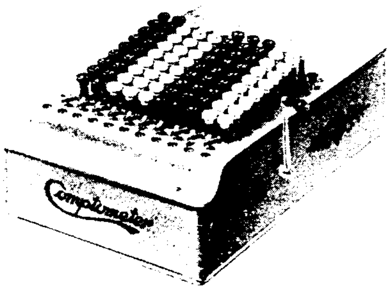
operated calculator with) to 13 columns. [*Burroughs*]



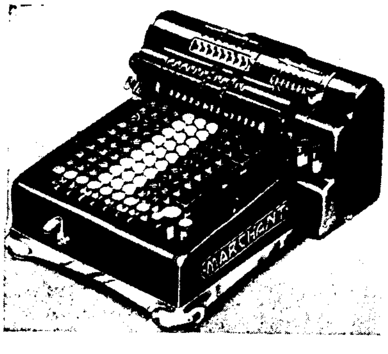
Direct adding, multiplying, subtracting, and dividing machine. Electrically or hand-operated by stop-and-start motor. Full automatic division. Standard flexible keyboard with locked-figure feature. Decimal markers on dials and keyboard. [*Monroe*]



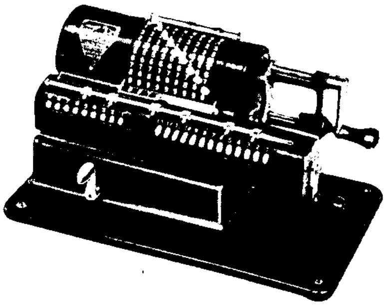
A compact adding-calculator weighing 13¼ pounds and occupying 6 × 8½ inches desk space. Direct adding, multiplying, subtracting, and dividing machine. Decimal markers on dials and keyboard. Electrically operated, alternating or direct current. May be operated as a hand-model machine by inserting the hand crank supplied. [*Monroe*]



A direct-acting adding, multiplying, dividing, and subtracting machine, performing every arithmetical calculation. [*Comptometer*]



An electrically operated calculator, performing additions, divisions, subtractions, and automatic multiplications and fractions. [*Marchant*]



A portable calculator, hand-operated. [*Marchant*]

Systematic procedures

Hartree's work shows how he made the connections between all the rows of numerical values more and more explicit. Even in his Numerical Analysis attention is shifting from reckoning to the design of schedules and procedures

values of f can be fitted by a polynomial of the nth degree. It can also be used to determine values of this polynomial for other values of x, and so to carry out interpolation.

The latter calculation can be done by a process of building up from nth-order divided differences, rather in the way in which a polynomial of the nth order can be built up, at equal intervals in x, from its nth differences (§ 4.42). Further, it is possible to determine derivatives of this polynomial, as follows.

If $x_{j+1} = x_j+\epsilon$, then $f(x_j, x_{j+1}) = f'(x_j)+O(\epsilon)$, and in the limit $\epsilon \to 0$, $f(x_j, x_j) = f'(x_j)$. Although $f(x_j, x_j)$ cannot be evaluated directly from the values of f and the definition (5.34) of divided differences, it can be built up from *higher* orders of divided differences and so determined in this way. Similarly

$$f(x_j, x_j, x_j) = \frac{1}{2!}f''(x_j)$$

and in general $$f(\underbrace{x_j, x_j, \ldots, x_j}_{n+1 \text{ arguments}}) = \frac{1}{n!}f^{(n)}(x_j).$$

Example: To show that the following values of $f(x)$ are consistent with $f(x)$ being a cubic in x, and to find $f(6), f'(6), f''(6)$ for this cubic:

x	-1	0	2	3	7	10
f	-11	1	1	1	141	561

The working can be arranged as follows:

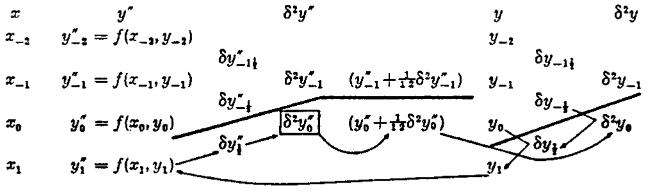
x	f	*1st order*	*2nd order*	*3rd order*
-1	-11			
		12/1 = 12		
0	1		$-12/3 = -4$	
		0/2 = 0		4/4 = 1
2	1		0/3 = 0	
		0/1 = 0		7/7 = 1
3	1		35/5 = 7	
		140/4 = 35		8/8 = 1
7	141		105/7 = 15	
		420/3 = 140		3/3 = 1
10	561		$-18/-1 = 18$	
		$-448/-4 = 122$		$-1/-1 = 1$
6	73		$-68/-4 = 17$	
		54		$-4/-4 = 1$
6			13	
6				

The working above the inclined line is concerned with showing that the third-order divided differences are constant, as is necessary for a cubic; that below the line is concerned with the evaluation of this cubic and its derivatives at $x = 6$. The arrows indicate the sequence in which the numbers in the lower part are obtained.

variants of it, some of which will be mentioned later. It will be supposed that the term $\frac{1}{240}\delta^4 y_0''$ in (7.2) is negligible, so that the integration formula is being used in the form

$$\delta^2 y_0 = (\delta x)^2[y_0''+\tfrac{1}{12}\delta^2 y_0''].\qquad (7.3)$$

Suppose the integration has reached $x = x_0$, and we are concerned with the integration through the interval δx to $x = x_1$. At this stage we have y_0 and $y_0'' = f(x_0, y_0)$ and the *backward* differences from these. The procedure is then as follows. Estimate $\delta^2 y_0''$, and obtain an approximation to $\delta^2 y_0$ from (7.3). Add this to $\delta y_{-\frac{1}{2}}$ to give an approximation to $\delta y_{\frac{1}{2}}$, and add this to y_0 to give an approximation to y_1. From this calculate y_1'' and hence $\delta^2 y_0'' = y_1''-2y_0''+y_{-1}''$. Let ϵ be the difference between this value of $\delta^2 y_0''$ and that estimated. A change of the estimate of $\delta^2 y_0''$ by ϵ makes a change $\frac{1}{12}(\delta x)^2\epsilon$ in y_1. If this is less than $\frac{1}{2}$ in the last figure retained in y, the estimate is adequate; if not, the estimate is revised and the calculation of the interval repeated; but the interval length (δx) should be taken so that this is seldom necessary. A convenient arrangement of the work is as follows:



The quantities above the heavy lines are those which are known when the integration has reached $x = x_0$; the quantity $\delta^2 y_0''$ enclosed in a 'box' is that which is estimated and if necessary adjusted, and the arrows show the sequence in which the various quantities are calculated.

To start the integration, two values of y are required, and it is advisable to have three to provide a check and to give an indication of the values of $\delta^2 y''$. These initial values will often be obtainable from a solution in series without requiring the evaluation of a large number of terms. In some cases it may be necessary to carry out a few steps of an integration at a small interval before starting the main integration.

Example: $y'' = (1-x^2)y$, $y(0) = 0$, $y'(0) = 1$, $\delta x = 0{\cdot}1$. $y(0{\cdot}1)$ and $y(0{\cdot}2)$ evaluated from the series solution:

$$y = x+\tfrac{1}{6}x^3-\tfrac{1}{24}x^5+O(x^7).$$

Programming

Edsger W. Dijkstra (*1930) and his teacher Aad van Wijngaarden clearly made the abstraction step from forms, schedules and computing procedures to programming

53

COMMUNICATION WITH AN AUTOMATIC COMPUTER

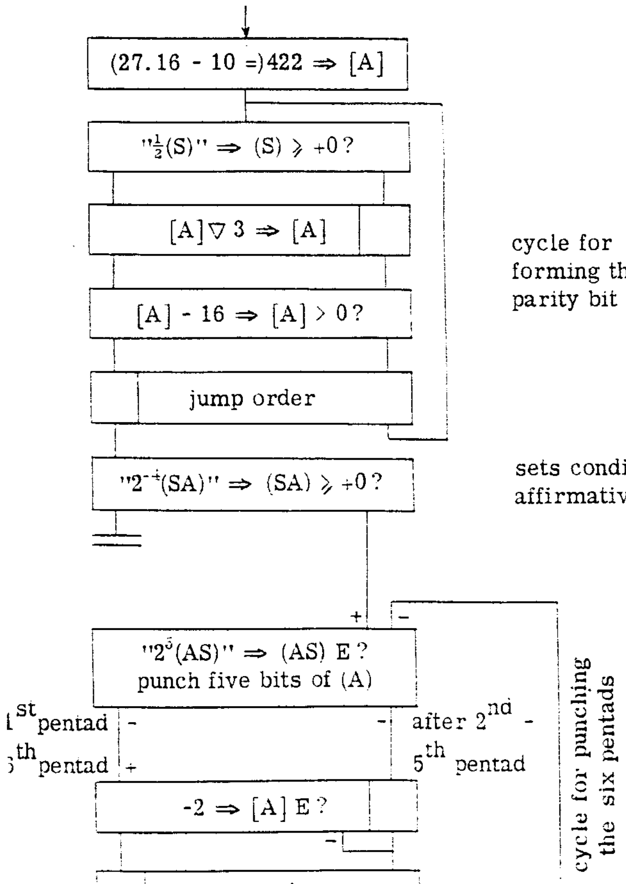
ACADEMISCH PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD VAN DOCTOR IN DE WIS- EN NATUURKUNDE, AAN DE UNIVERSITEIT VAN AMSTERDAM, OP GEZAG VAN DE RECTOR MAGNIFICUS DR. J. KOK, HOOGLERAAR IN DE FACULTEIT DER WIS- EN NATUURKUNDE, IN HET OPENBAAR TE VERDEDIGEN IN DE AULA DER UNIVERSITEIT OP WOENSDAG 28 OCTOBER 1959 DES NAMIDDAGS TE 3 UUR PRECIES

DOOR

EDSGER WYBE DIJKSTRA

GEBOREN TE ROTTERDAM



Exit Numerical Analysis

With the focus of programming ends the story of Numerical Analysis. The emerging trade exploded into a broad spectre of new subjects, the discipline of programming (computer science, software engineering, informatics) being one of them. Among the other parts were scientific computing, numerical mathematics, systems development and computational branches of the engineering disciplines.

The gain of this development was a clearcut focus on one systematic problems. Now a true discipline did emerge.

The loss of the explosion of Numerical Analysis was the orientation towards solving real world problems. In computer science one hardly worries about the mathematical modelling that necessarily preceeds the application of computing to real problems.

1.4. Arrangement of work

In most numerical work, a working sheet will be used for recording data and intermediate results of the calculation. A clear and orderly arrange-

ment of this working sheet is a great help both in avoiding mistakes and in locating and correcting any that do happen to be made. Numerical work should not be done on odd scraps of rough paper, but laid out systematically and in such a way as to show how the intermediate and final results were obtained; and the numbers entered on the work sheet should be written neatly and legibly. Use of ruled paper is a help in keeping the layout of the work neat and clear. It is advisable to use loose sheets rather than a book since it is rather easy to make mistakes in copying from one page to another of a book; with loose sheets the number to be copied from one sheet, and the place to which it is to be copied on another, can more easily be brought close together, and the copy made and checked more easily.

For work of any permanent value, it is advisable to record on the working sheet enough explanation of the different entries, and how they were obtained, for the working to be followed after the lapse of a period of years.

1.5. Accuracy and precision

In contexts in which numerical work is carried out in connexion with scientific and technical problems, we are often concerned with the numerical solution of one or a set of algebraic, differential, or integral equations. Then it may be convenient to distinguish between the accuracy to which the equations, or data used in obtaining a solution of them, represent the real situation to which they refer, and the accuracy to which the results of the numerical work represent the solution of these equations with these data, supposed exact. The latter is sometimes distinguished by being called the 'precision' or 'nominal accuracy' of the numerical work.

Calculations are often carried out deliberately to a nominal accuracy known or expected to be higher than the accuracy of the approximations made in deriving the equations, or higher than that of the data used in their solution. There are several reasons why this may be done. We may be interested in the differences between the results of observation and

The USE of history

Didactics

Recognizing old practices may and the genesis of todays knowledge may enhance the understanding of present practices and hence ease the learning by the students. Hartree's practices are easily compared with spreadsheet packages introduced in the 1980's.
The silently assumed parallel between historical genesis and the psychological process of learning needs some caution.
History used in this way tends to be whiggish.

Self reflection

Training academics who are able to see the power and limitations of their discipline is enhanced by such refelxive courses as history of the discipline. Confrontation with a different discipline, viz. history, will make the students think.
Looking at different practices in different times and circumstances will allow the students to look further then the self-evidences of today's computer science. If it was not always so, then it is not necessarily so. This will produce wiser computer scientists.

ZEITSCHRIFT FÜR ANGEWANDTE MATHEMATIK UND MECHANIK

INGENIEURWISSENSCHAFTLICHE FORSCHUNGSARBEITEN

Band 7 **Juni 1927** **Heft 3**

Inhalt:

HAUPTAUFSÄTZE

Beiträge zur Lösung des ebenen Problems eines elastischen Körpers mittels der Airyschen Spannungsfunktion.[1]

Von **WOLFGANG RIEDEL** in Göttingen.

(Mitteilung aus dem Institut für angewandte Mechanik der Universität Göttingen.)

Das ebene Problem eines elastischen Körpers läßt sich bekanntlich mit Hilfe der Airyschen Spannungsfunktion behandeln[2]). Als Bezeichnungsweise der elastischen Größen ist folgende gewählt: ξ, η, ζ bedeuten die Komponenten der Verschiebung eines Punktes (x, y; z), σ_x, σ_y, σ_z die Normalspannungen, τ_{xy}, τ_{xz}, τ_{yz} die Schubspannungskomponenten, ε_x, ε_y, ε_z die spezifischen Dehnungen, γ_{xy}, γ_{xz}, γ_{yz} die spezifischen Schiebungen